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2805 [1920, 32]. **Proposed by C. N. MILLS, Brookings, S. Dakota.**

Derive the expression for volume

$$v = \int\int\int \rho^2 \sin \phi d\rho d\phi d\theta.$$

In Byerly's *Integral Calculus*, page 183, revised edition, is a method of revolution, and in Czuber's *Integralrechnung*, page 200, is a method using the Jacobian determinant.

Required, a simple method one might use in developing the volume integral in polar coordinates.

SUGGESTION BY PAUL CAPRON, U. S. Naval Academy.

Draw a figure to show an octant of a sphere, as it might be of the earth with the North Pole atop, from Long. 0° to Long. 90° W. and from Lat. 0° to Lat. 90° N. The angle θ will correspond to the longitude, ϕ to the complement of the latitude. Show two meridians $d\theta$ apart and two parallels $d\phi$ apart. On the rectangular area so bounded build up an element of volume by extending through its corners four radii, projecting a distance $d\rho$ to a concentric spherical surface, on which they mark the corners of another rectangular base.

The sides of the inner rectangular base are $\rho \sin \phi d\theta$, $\rho d\phi$, $\rho \sin (\phi + d\phi)d\theta$, $\rho d\phi$; the sides of the outer base are the same, with $(\rho + d\rho)$ in place of ρ.

This point in the discussion should be reached at just about the end of hour. Conclude by leaving to the individual students the task of proving, each to his own satisfaction, that the neglected infinitesimals are of order higher than the first. Serve out enough applications to furnish a diversion of interest and start something useful and fascinating at the next lecture.

Also answered by T. M. BLAKSLEE, A. R. NAUER, H. L. OLSON, D. H. RICHERT, and ELIJAH SWIFT.

2806 [1920, 32]. **Proposed by R. E. MORITZ, University of Washington.**

An anthropologist told me recently that large numbers of Russian peasants, whose knowledge of numbers is limited to multiplication and division by 2, employ the following method of multiplication which they were taught by a priest.

(1) Write the two numbers to be multiplied in the same horizontal line.

(2) Multiply the first number by 2, and write the product under the number so multiplied.

(3) Divide the second number by 2, discarding the remainder 1 when it occurs, and write the quotient under the number so divided.

(4) Treat the product and quotient thus obtained in the same manner as the original numbers. Continue this process until the quotient 1 is obtained.

(5) Strike out all the numbers on the left for which the corresponding numbers on the right are even.

(6) Add the remaining numbers on the left. Their sum is the required product.

Problem: Prove that this rule is correct.

SOLUTION BY P. R. RIDER, Washington University.

The method depends on the fact that any number can be written in the binary scale of notation, that is, as a sum of positive integral powers of 2. To express a number in this way we divide successively by 2 until the quotient is 1. This 1 will be the first digit, beginning at the left, and the remainders (always either 1 or 0) will be the other digits expressing the number. That is, the remainder after the rth division by 2 will be the digit in the rth place, counting from the right, in the binary scale expression of the number, or the coefficient of 2^{r-1} in the expansion of the number in positive integral powers of 2. (See Todhunter's *Algebra for the use of Schools and Colleges*, chapter 29.) Thus, the multiplication process considered consists essentially in expressing one number in positive integral powers of 2 and multiplying the other number by this expression.

For instance, suppose that the numbers to be multiplied by the method are a and b, a heading the first column and b the second. Then the second column is the work of developing b in powers of 2, and the numbers in the first column are the terms of this development, each multiplied by a. For, if a number in the rth line of the second column is odd, there will be a remainder of 1, and the corresponding digit in the expression of b in the binary scale is 1, giving the term of value $1 \times 2^{r-1}$ in the development of b in powers of 2. If, on the other hand, the number in the rth line of the second column is even there will be no remainder, and the corresponding digit in the binary scale expression of b is 0, giving the term of value $0 \times 2^{r-1}$ in the development of b. But the number in the rth line of the first column is $2^{r-1} \times a$, and since all numbers in the first column that are opposite even numbers in the second column are stricken out, the sum of the remaining numbers will be precisely $b \times a$.

A numerical example will make this much clearer. The work for the multiplication of 14 and 83 would appear (except for the figures in parentheses) as follows:

$$\begin{array}{rr}
(2^0 \times 14 =)^{*}\ 14 & 83 \\
(2^1 \times 14 =)^{*}\ 28 & 41 \\
(2^2 \times 14 =)\ 56 & 20 \\
(2^3 \times 14 =)\ 112 & 10 \\
(2^4 \times 14 =)^{*}224 & 5 \\
(2^5 \times 14 =)\ 448 & 2 \\
(2^6 \times 14 =)^{*}\underline{896} & 1 \\
1162 &
\end{array}$$

The number 83 expressed in the binary scale of notation would be 1010011 (*i.e.*, $83 = 2^1 + 2^4 + 2^1 + 2^0$). Thus the sum of those parentheses marked with an asterisk is $(2^6 + 2^4 + 2^6 + 2^0) \times 14$, or 83×14.

In presenting a similar discussion Professor U. G. Mitchell cited the article in this Monthly, *1918*, 139–142, by Professor R. C. Archibald, entitled: "The binary scale of notation, a Russian peasant method of multiplication, the game of nim, and Cardan's rings." Many references are there given to the literature of the history and discussion of the binary scale and its applications.—Editors.

Also solved by T. M. Blakslee, B. A. Bernstein, Paul Capron, Carl Gundersen, W. H. Hays, A. M. Kenyon, Theodore Linqduist, Rosco Lamont, H. F. MacNeish, L. C. Mathewson, H. L. Olson, Arthur Pelletier, W. B. Pierce, D. H. Richert, H. S. Uhler, and C. C. Wylie.

2822 [1920, 185]. Proposed by A. M. HARDING, University of Arkansas.

Show that the sum of the series:

$$1 + 3\cdot 2 + 5\cdot 2^2 + 7\cdot 2^3 + \cdots + (2n-1)2^{n-1}$$

(to n terms) is $3 - 2^n + (n-1)2^{n+1}$.

Solution by Louis O'Shaughnessy, Virginia Polytechnic Institute.

Set

$$S = 1 + 3\cdot 2 + 5\cdot 2^2 + 7\cdot 2^3 + \cdots + (2n-3)2^{n-2} + (2n-1)2^{n-1}.$$

Then

$$2S = 2 + 3\cdot 2^2 + 5\cdot 2^3 + \cdots + (2n-5)2^{n-2} + (2n-3)2^{n-1} + (2n-1)2^n.$$

Hence,

$$S = -1 - 2\cdot 2 - 2\cdot 2^2 - 2\cdot 2^3 - \cdots - 2\cdot 2^{n-2} - 2\cdot 2^{n-1} + (2n-1)2^n,$$

$$= -1 - \sum_{2}^{n} 2^n + (2n-1)2^n = -1 - (2^{n+1} - 4) + (2n-1)2^n = 3 - 2^n + (n-1)2^{n+1}.$$

Also solved by T. M. Blakslee, H. N. Carleton, P. J. Da Cunha, E. B. Escott, R. M. Ginnings, H. Halperin, Harry Levy, L. C. Mathews, H. L. Olson, Arthur Pelletier, A. V. Richardson, Etheldred A. Willmott, and C. C. Wylie.